

AN INTERPRETATION OF DUAL THEORIES

THY-7

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A possible connection of dual models with conventional field theories is presented.

In this short note, we present an interpretation of dual resonance models which may elicit their relation to conventional models.

To each point (x_μ, p_μ) in Lorentz phase space we associate an internal space in which motion is generated by the Hamiltonian function¹

$$H = \frac{1}{2} \sum_{m=0}^{\infty} (p^{(m)}_\mu \cdot p^{(m)}_\mu + \omega_m^2 q^{(m)}_\mu \cdot q^{(m)}_\mu) \quad (1)$$

in which the frequencies obey

$$\omega_{m+1} - \omega_m = \omega, \quad m = 1, 2, \dots$$

and

$$[q^{(m)}_\rho, q^{(n)}_\sigma] = [p^{(m)}_\rho, p^{(n)}_\sigma] = 0; \quad [q^{(m)}_\rho, p^{(n)}_\sigma] = i g_{\rho\sigma} \delta^{mn}; \quad (2)$$

here $g_{\rho\sigma} = (-1, +1, +1, +1)$ is the Lorentz metric.

This internal system has a total momentum

$$P_\mu = \sum_{m=0}^{\infty} p^{(m)}_\mu \quad (3)$$

corresponding to a coordinate

$$Q_\mu = \sum_{m=0}^{\infty} q^{(m)}_\mu \quad (4)$$

We call these generalized momentum and coordinate, respectively, and consider them as generalizations of the usual canonical coordinates on which conventional theories are constructed.

The Heisenberg equations of motion

$$P_{\mu}(\tau) = e^{i\tau H} P_{\mu} e^{-i\tau H} \quad (5a)$$

$$Q_{\mu}(\tau) = e^{i\tau H} Q_{\mu} e^{-i\tau H} \quad (5b)$$

allow us to introduce the variable τ which acts as a time variable for the internal system. For the physical limit $\omega_0 \rightarrow 0$, however, all the frequencies of the internal normal modes become integer multiple of ω , thereby enabling us to consider τ as a cyclic variable. The interval

$$-\frac{\pi}{\omega} \leq \tau \leq +\frac{\pi}{\omega} \quad (6)$$

defines the fundamental cycle of the internal system.

Then, all physical quantities are assumed to be the average over the fundamental cycle of the system of their generalized counterparts in the internal space. The average of an operator $A(\tau)$ is given by

$$\langle A(\tau) \rangle \equiv \frac{\omega}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} d\tau A(\tau) \quad (7)$$

It then follows that

$$\langle P_{\mu}(\tau) \rangle = p_{\mu}^{(0)} \equiv p_{\mu} \quad (8a)$$

$$\langle Q_{\mu}(\tau) \rangle = q_{\mu}^{(0)} \equiv x_{\mu} \quad (8b)$$

are the physical momenta and coordinates, respectively.

This procedure can be carried over to products of operators with the added proviso that these be normal ordered, except for the lowest mode.

Hence, the square mass of the physical system is given according to the correspondence principle

$$p_{\mu} p_{\mu} = \langle P_{\mu}(\tau) \rangle \langle P_{\mu}(\tau) \rangle \longrightarrow : \langle P^2(\tau) \rangle : \quad (9)$$

so that the generalization of the Klein-Gordon equation for a dual system is

$$\{ : \langle P^2(\tau) \rangle : + m^2 \} \phi = 0 \quad (10)$$

Use of the Heisenberg equations of motion and of the definition of the average yields, in the physical limit $\omega_0 \rightarrow 0$,

$$\{ p_{\mu} p_{\mu} + \omega \sum_{n=1}^{\infty} n a^{(n)\dagger}_{\rho} a^{(n)}_{\rho} + m^2 \} \phi = 0 \quad (11)$$

which is the well-known form found in the Veneziano model². Here $a^{(n)\dagger}_{\rho}$ and $a^{(n)}_{\rho}$ are the creation and annihilation operators, respectively. These are given by

$$a^{(n)\dagger}_{\rho} = \frac{1}{\sqrt{2}} \left\{ \sqrt{\omega_n} q^{(n)}_{\rho} - \frac{i}{\sqrt{\omega_n}} p^{(n)}_{\rho} \right\} \quad (12a)$$

$$a^{(n)}_{\rho} = \frac{1}{\sqrt{2}} \left\{ \sqrt{\omega_n} q^{(n)}_{\rho} + \frac{i}{\sqrt{\omega_n}} p^{(n)}_{\rho} \right\} \quad (12b)$$

The solutions of the generalized Klein-Gordon equation will in general contain ghost states which have been introduced by the Lorentz metric.

We must therefore establish subsidiary conditions that will insure their absence in the solutions of Eq. (10). These are conventionally written as

$$p_{\mu} a_{\mu}^{(n)} | \phi \rangle = 0 \quad n = 1, 2, \dots \quad (13)$$

which can be rewritten as

$$\langle P_{\rho}(\tau) \rangle \langle e^{i\omega_n \lambda} P_{\rho}(\lambda) \rangle | \phi \rangle = 0 \quad n = 1, 2, \dots \quad (14)$$

However, according to our correspondence principle, these become

$$: \langle e^{i\omega_n \lambda} P_{\rho}(\lambda) P_{\rho}(\lambda) \rangle : | \phi \rangle = 0 \quad n = 1, 2, \dots \quad (15)$$

In terms of creation and annihilation operators, they read, in the physical limit $\omega_0 \rightarrow 0$,

$$\left\{ i\sqrt{2n} p_{\mu} a_{\rho}^{(n)} + \frac{\sqrt{\omega}}{2} \sum_{\ell=1}^{n-1} \sqrt{\ell(n-\ell)} a^{(\ell)} \cdot a^{(n-\ell)} - \sqrt{\omega} \sum_{\ell=1}^{\infty} \sqrt{\ell(\ell+n)} a^{(\ell)\dagger} \cdot a^{(\ell+n)} \right\} | \phi \rangle = 0 \quad (16)$$

$n = 1, 2, \dots$

These are seen to reduce to the equations found to hold by Virasoro³ in the special case of unit intercept in the Veneziano model.

Other operators of interest in the classification of the solutions of the generalized Klein-Gordon equation are the Lorentz generators which we obtain by means of the correspondence principle

$$\epsilon_{\mu\nu\rho\sigma} x_{\rho} p_{\sigma} = \epsilon_{\mu\nu\rho\sigma} \langle Q_{\rho}(\tau) \rangle \langle P_{\sigma}(\tau) \rangle \rightarrow : \epsilon_{\mu\nu\rho\sigma} \langle Q_{\rho}(\tau) P_{\sigma}(\tau) \rangle : \quad (17)$$

Their explicit expression is easily derived⁴

$$-i\epsilon_{\mu\nu\rho\sigma} \left[ix_{\rho} p_{\sigma} + \sum_{m=1}^{\infty} a_{\rho}^{(m)\dagger} a_{\sigma}^{(m)} \right] \quad (18)$$

We recognize in this expression the orbital and spin parts, which insures us that the solutions of the Klein-Gordon equation can have arbitrarily high integral spin.

Although it is difficult to construct the most general solutions of our Klein-Gordon equation which obey the subsidiary conditions (15), it is clear, in analogy with the usual free field theories, that these should be expressed in terms of the super plane waves

$$e^{iP_{\mu}(\lambda) Q_{\mu}(\lambda')}$$

since

$$\left[Q_{\mu}(\lambda), P_{\rho}(\lambda') \right] = \frac{1}{2} g_{\rho\mu} \left(\delta \left(\frac{\lambda - \lambda'}{\omega} \right) + 1 \right) \quad (19)$$

We point out, however, that Q 's do not commute together at different τ 's, which renders the solution of (10) difficult.

The form of the three Reggeon vertex⁵ obtained through factorization of the Veneziano amplitude points out to a system in interaction of the form

$$\{ :<P^2(\tau) > : + m^2 \} \phi = \lambda \phi^2 \quad (20)$$

The Veneziano amplitude would then be the Born term of this new field theory in the case of scalar external states.

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